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AN OVERVIEW OF GEOMAGNETIC FIELD MODELS

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PREFACE

The Phoenix Corporation has conducted an extensive review of the Department of Defense Geomagnetics Program. While the entire program has been discussed in a separate volume, it was decided to address geomagnetic field models separately as they are a key program element.

Over the last decade, the utility of geomagnetic field models or the mathematical representations of the geomagnetic field has expanded considerably, both for civilian and military applications. Such an increase has been accompanied with more interest in models and demands for better accuracies with resultant changes in techniques of model computations and input data requirements. Thus, it is also timely to review this area of geomagnetic research. Additionally, such a detailed overview of geomagnetic field models provides a vantage point for reviewing the role of Project MAGNET and provides assessment of the general utility of field models.

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INTRODUCTION

The study of the Earth's magnetic field is one of the oldest subjects in man's quest for knowledge about his environment. It has long been known that the Earth's magnetic field behaves somewhat similar to that of a giant bar-magnet lying generally in a north-south direction, located near the Earth's rotational axis. Indeed, such a description of Earth's magnetic field was put forth by Sir William Gilbert (1540-1603) in his book De Magnete which started the field of geomagnetism. Continuing investigations on the nature of Earth's magnetic field, however, have indicated that the field is more complex than that assumed by Gilbert. For example, Gauss (1839) in his spherical harmonic analysis of the geomagnetic field showed that an accurate description of the field required many more parameters than the magnitude and orientation of a simple bar-magnet (dipole) approximation. Furthermore, he showed that only 94% of the observed field could be accounted for by sources inside the Earth. Also, there are irregular spatial variations of the field over the surface of the Earth, and irregular temporal variations in the field have been noted since as early as 1634. As a result of such studies over the past few centuries, it is now known that the Earth's magnetic field is composed of three parts:

a) The 'main field' or the internal field, which although not constant in time, varies relatively slowly (usually described in terms of years) and originates in the Earth's core.

b) The 'external field,' a relatively small fraction of the amplitude of the main field which varies rather rapidly, partly cyclically and partly randomly, and which originates outside the solid Earth in the ionospheric and magnetospheric regions.

c) 'Crustal anomalous field,' smaller than the main field, relatively constant in time and place and the resultant of local magnetic anomalies in the near-surface crust of the Earth.

As the magnetic field is a vector quantity, a vector sum of the above three component fields is recorded in any geomagnetic measurement. Of all the three components, however, the main field is the dominant part of the geomagnetic field. This field varies continuously over the Earth's surface, with intensity ranging from about 25,000 gammas to 70,000 gammas. Its relative and predictable smoothness is what makes it ideally suited for applications such as navigation.

Since the magnetic field is a vector quantity, it is necessary to measure three of the seven conventional geomagnetic elements (Figure 1) in order to specify the field completely. In this diagram, X, Y, and Z are the three mutually orthogonal components of the total field vector $\overline{F}$, with the X axis pointing towards geographic north. $\overline{H}$ is projection of $\overline{F}$ in the horizontal plane, and D and I are known as declination and inclination respectively. Lines of equal inclination, declination, total field intensity, etc., when plotted on maps are called 'isomagnetic lines' and represent the variations in the geomagnetic field over the Earth's surface. Such charts for the F, I and D elements of the field are shown in Figures 2, 3 and 4 respectively. Strikingly, these charts show little or no relation to changes in surface geology and geography such as mountain ranges, submarine ridges and trenches, earthquake belts and thus it is indicated that the source of the field lies deep within the Earth's interior.

Though, as has been noted, the Earth's main field varies smoothly over the surface, it is far from permanent in time as geomagnetic field reversals over geologic history are well documented. These temporal changes in the main

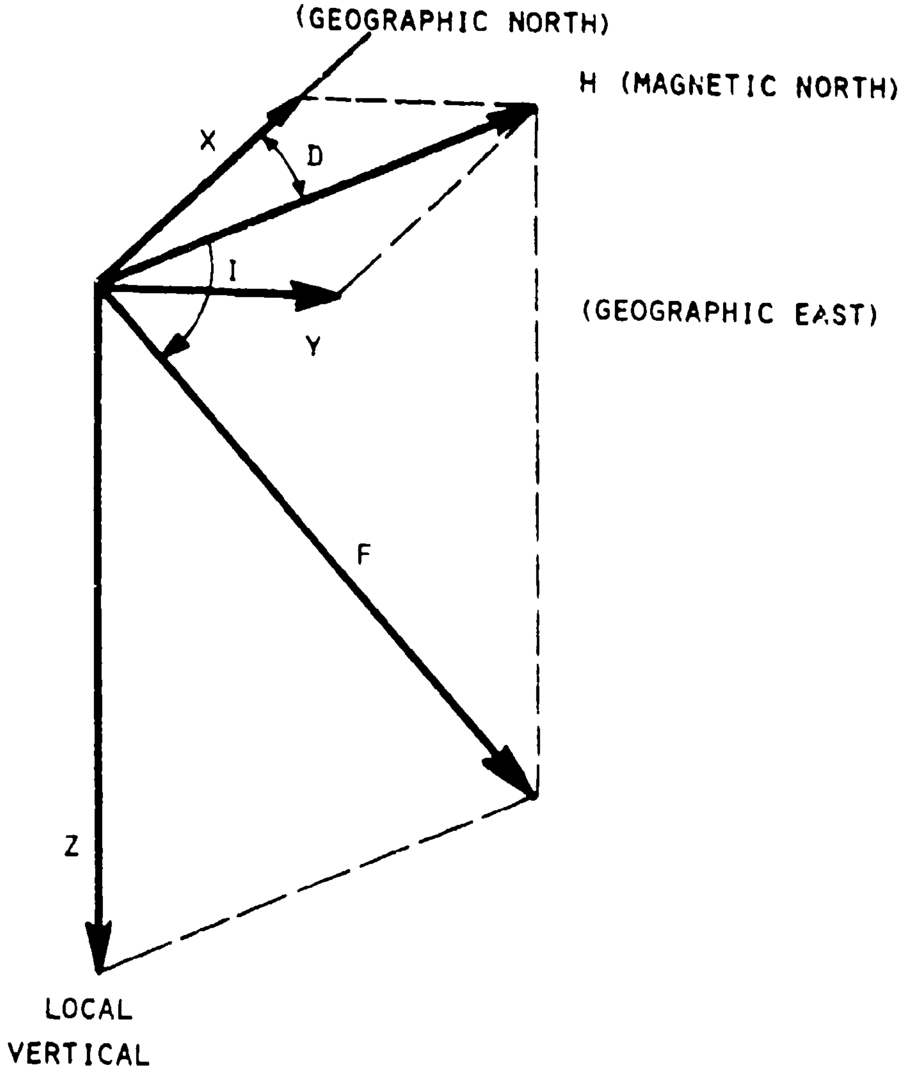

Figure 1. Relationship of geomagnetic field elements.

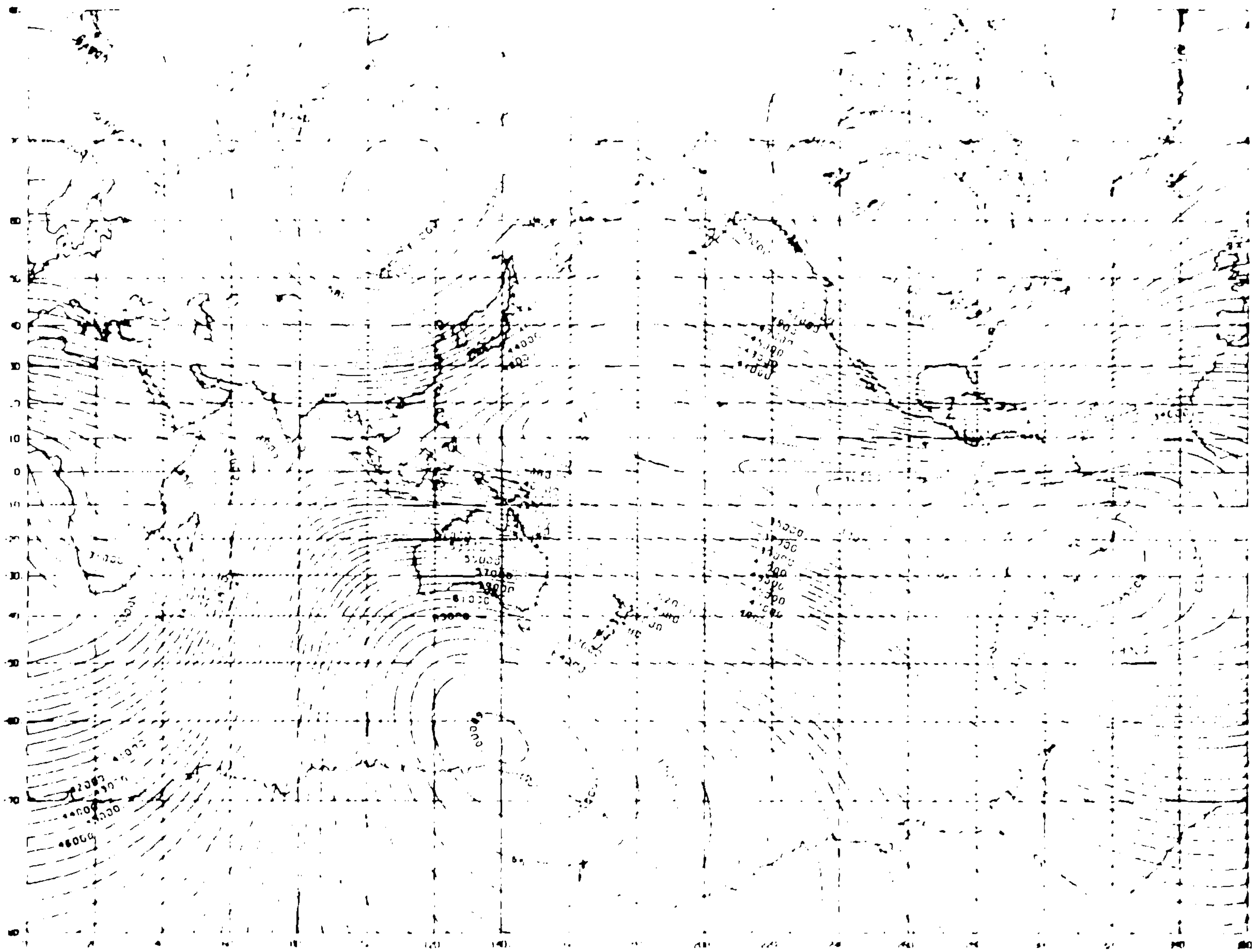

Figure 2. Contours of Constant F (Total Field) for World Magnetic Chart Model 1980.0.

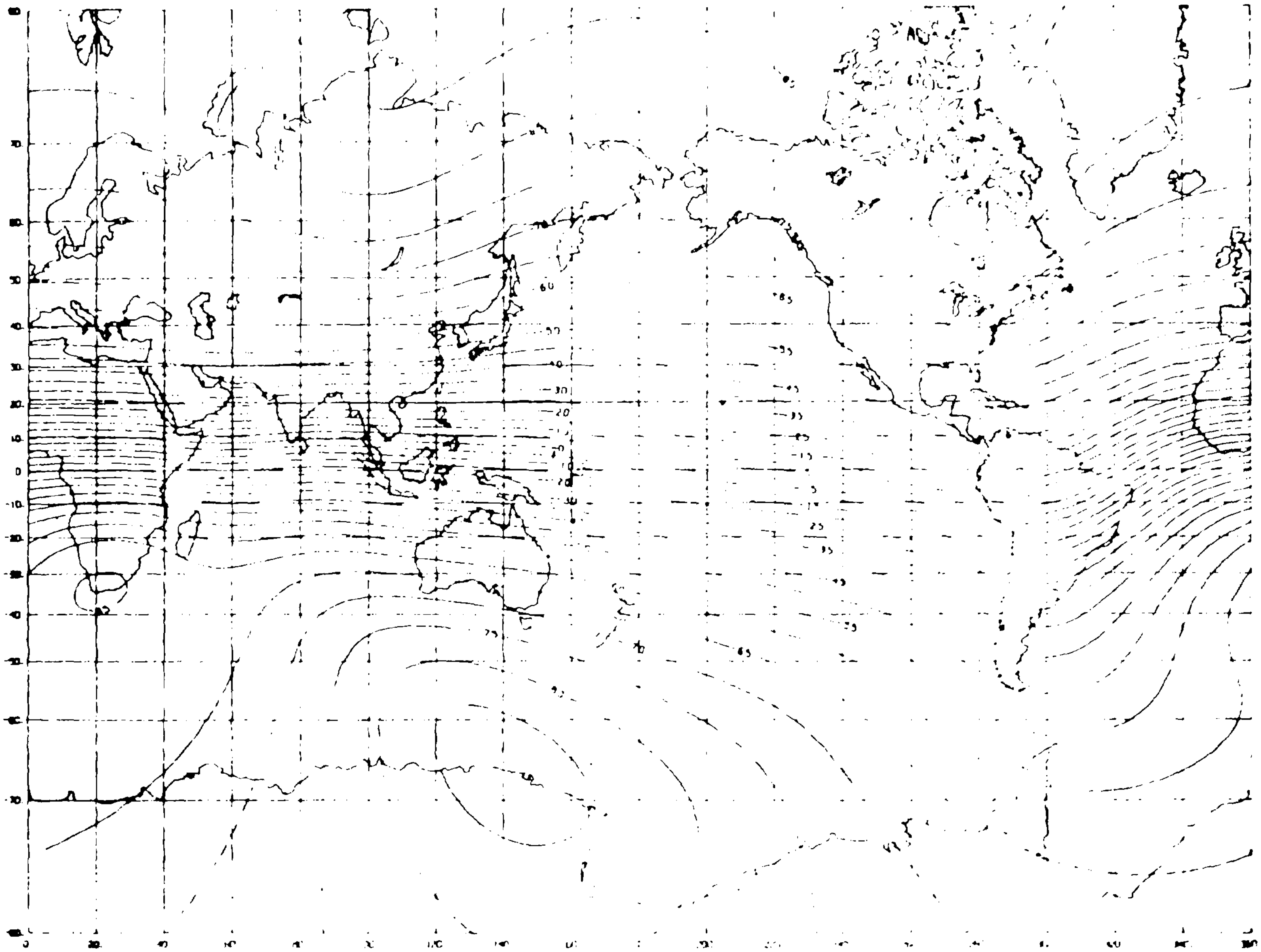

Figure 3. Contours of Constant I (Inclination) for World Magnetic Chart Model 1980.0.

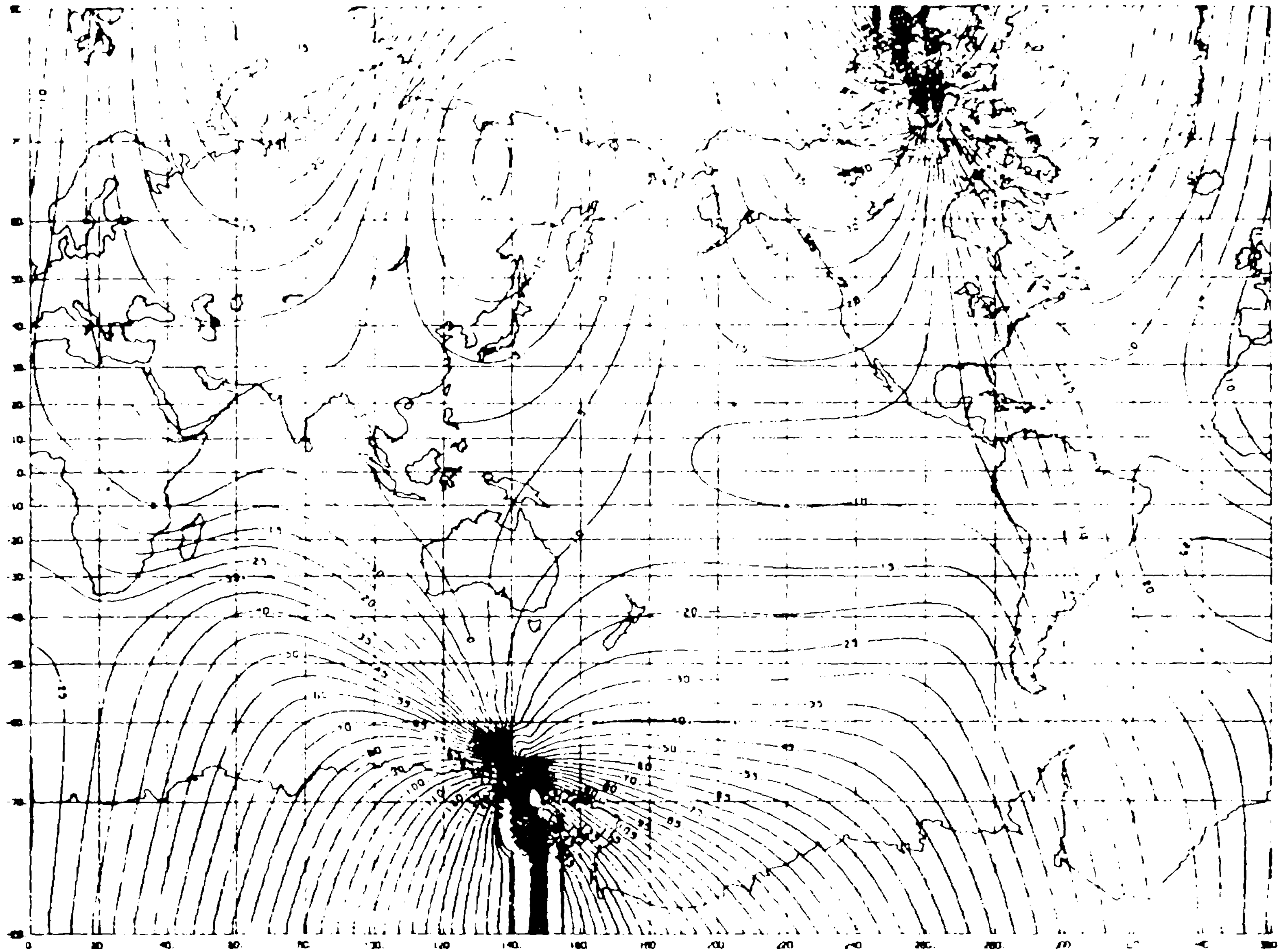

Figure 4. Contours of Constant D (Declination) for World Magnetic Chart Model 1980.0.

field are viewed as long term phenomena and are usually measured over a period of years. Collectively such temporal changes in the main field are called the secular variation. From the recent analysis of MAGSAT data, Langel et al (1980) have shown that the Earth's dipole field is declining at a rate of 26 gammas/year and if the present trend continued, the Earth's field would decrease to zero within 1000 years. Such secular variation, however, is neither constant over time nor is it distributed uniformally over the earth's surface. In all cases, these secular variations appear to be regional rather than worldwide. As for the isomagnetic maps, the rate of the secular variation in the various elements of the magnetic field can also be represented in a map form termed isoporic charts. An example of such a chart, the secular variation of the total field intensity, F, is shown in Figure 5.

Though the geomagnetic field has been studied for centuries, the underlying cause of it is far from certain. However, the most commonly accepted theory is that of a self-exciting dynamo, a concept originally proposed by Larmor in 1919 and later developed by Bullard in 1949. The Earth's outer core, which extends from a depth of 2900 km to 5100 km, is known to have the properties of a liquid from seismic evidences. For several reasons, it is assumed that this material is a combination of iron and nickel, both good conductors of electricity. The self-exciting dynamo theory suggests that the internal field is maintained by electric currents within the Earth induced by the conducting material in the core which is set into motion by convection. The secular variation of the main field can also be explained by this theory as it is most likely connected with the changes in the convection currents in the core, in the core mantle coupling, and in the rotational speed of the Earth.

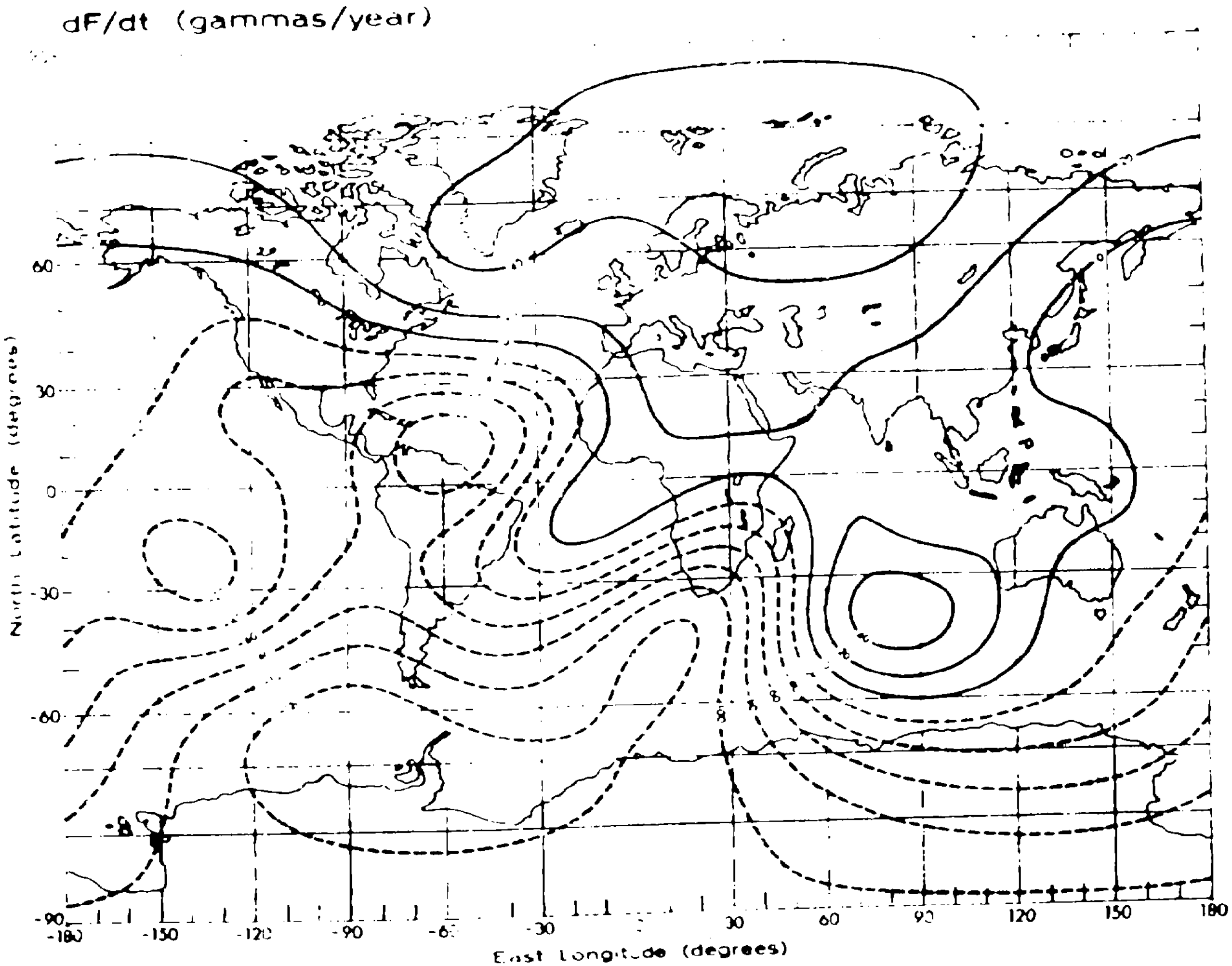

Figure 5. Contours of Constant Secular Change in F (Total Field) for Model IGRF 1965.0.

The external part of the geomagnetic field, the external field, originates from sources outside the Earth and comprises roughly 1% of the total field observed at the surface. Included in this is the field induced in the Earth's crust due to the time variability of the external field sources. The variation in time of this field is much more rapid than the secular variation of the Earth's main field. Numerous studies of this field have well established that these are associated with the electric currents in the ionospheric and magnetospheric regions. Though the external fields are highly erratic and range in amplitude from several milli-gammas to hundreds of gammas, some of the well-documented effects are listed below.

i) An 11-year cycle of variations in the field intensities are noted which correlates well with the sunspot activities and has a latitude-dependent distribution.

ii) Diurnal variations, which have a periodicity of about a day and amplitude averaging about 25 gammas. These variations vary with latitude and season, and are controlled by action of the sun on the ionospheric currents. In general, two types of variations are noted: the 'quiet day' variation (Sq) and the 'disturbed day' variation (S_D). The quiet day variation is smooth, regular and low in amplitude whereas the disturbed day variation is less regular, higher in amplitude and is associated with magnetic storms.

iii) Lunar diurnal variation (L) with a periodicity of approximately 25 hours and having amplitudes about one-fifteenth the amplitude of the solar Sq diurnal variation. These vary cyclically through the month and are associated with lunar gravitational influence on the ionosphere.

iv) Magnetic storms which are transient disturbances lasting for several days and having amplitudes as large as 1000 gammas in most latitudes and even greater in polar regions, where they are usually accompanied by aurora. Magnetic storms are usually not predictable, but they tend to come at intervals of 27 days, a frequency correlating with the rotation of the sun.

A more detailed breakdown of the general characteristics of external magnetic fields is given in Table 1 which lists the various types of external field phenomena observed over the Earth's surface, its typical period and amplitude and its probable origin.

The third component of the geomagnetic field is the crustal anomalous field which arises as a result of the variations in the magnetic mineral content of crustal rocks. Such fields are essentially time invariant and local in nature and are also called local magnetic anomalies. The magnitude of these anomalies is typically on the order of several hundred gammas and may extend from one to several hundred square kilometers in areal coverage. Such magnetic anomalies are the primary targets of exploration geophysical magnetic surveys. The depths to the sources of these anomalies are presumed to be less than 20 km, as the Curie Point temperature is exceeded below this depth. Thus, the crustal anomalous fields are primarily associated with the near-surface features.

It can thus be summarized that the Earth's magnetic field characteristically arises from two classes of sources, one internal to the Earth and the other external to it. The part internal to the Earth, is a relatively slowly time varying field with time constants of tens to thousands of years and can thus be regarded as a permanent or steady field for many applications. This steady part of the Earth's total field is also known as the main field and it results primarily from convective motion in the core. Approximately 90% of the main field is dipolar in nature, with the ideal dipole centered close to the geographic center of the Earth and its axis, inclined about 11.5 degrees to the

TABLE 1

GENERAL CHARACTERISTICS OF EXTERNAL MAGNETIC FIELD TEMPORAL VARIATIONS

Type	Origin	Typical Amplitude* (Gamma)	Typical Period*	Comments
		Quiet Variations		
Sq	Solar electromagnetic radiation and gravitational influence on ionosphere	tens	several hours	- main variations during day - correlated with local time - abnormally high near dip equator (equatorial electrojet) - greater in summer than in winter - subtle variation with IMF direction
L	Lunar tidal influence on ionosphere	several	several hours	- main variations during day - varies with lunar phase and season - abnormally large over dip equator-greater magnification than Sq
		Disturbed Variations		
Magnetic Storm	Interaction of solar plasma with magnetosphere	several hundred	several days	
- initial phase	Magnetospheric compression		hours	- abrupt increase in magnetic field - highly irregular
- main phase	- enhanced ring current - combined effects of magnetospheric substorms-release of energy from tail		1 - 2 days	- gradual decrease below pre-storm level
- recovery phase	- decay of ring current		hours - days	- gradual return to pre-storm level

TABLE 1 (Cont'd)

Type	Origin	Typical Amplitude* (Gamma)	Typical Period*	Comments
Magnetic bays	- auroral electrojet (DP1) - magnetospheric substorms	hundreds	several hours	- mostly an auroral zone phenomena - effect can extend over 100° in longitude from source - intensity of onset depends on proximity to source - maximum intensity near geomagnetic midnight
S	- polar ionospheric current	tens	several hours	- auroral zone phenomena analagous to Sq
DP2	- polar ionospheric current	several	minutes	- global phenomena
Zone of high latitude field agitation	?			- occurs in polar cap area near local noon (except winter)
DPC	- IMF By			- polar cap phenomena
Crochet (solar flare effect)	- enhancement of ionization by solar flare x-ray emission			- apparent sudden amplification of Sq - increase in equatorial electrojet effect
Micropulsations				
- pc	- ionospheric magnetospheric perturbations	several	2 - 40 sec.	- classified into subgroups - diurnal - maximum near local noon
- pt	- related to polar disturbances	several	2 min - 1 hour	- nocturnal - related to magnetic bay
- pp	- hydromagnetic emissions	several	0.2 - 5 sec.	- amplitude modulated

*Amplitudes and periods of external magnetic field temporal variations are highly variable and depend on their physical origin as well as the geomagnetic latitude, longitude, and time of the observation point. Values given here are typical.

axis of rotation. The remaining 10% of the main field, often termed the residual field, is nondipolar. It consists of both large-scale anomalies (up to thousands of kilometers) believed to be generated by eddy currents in the fluid core, and small-scale irregularities (up to tens of kilometers) originating from residual or induced magnetism in the Earth's crust. The part of the Earth's magnetic field, originating from outside the Earth, is called the external field and recent analyses have indicated it to be less than 1% of the total field measured at the Earth's surface.

Historically, the use of the Earth's magnetic field has been in the field of navigation relying on the directive action of the field upon compass needles. For such purposes, declination maps were prepared by hand by drawing smooth curves through the measured data available from permanent observatory locations and periodic ship measurements. However, as the knowledge about the Earth's field and its applications in other areas of physics widened, it became increasingly apparent that such a chart or map representation of the field was not adequate. For example, in space sciences applications, it is necessary to know the Earth's field at satellite altitudes and the charts which represent field values measured at the Earth's surface are of little use. Also, in magnetic surveys for mineral or petroleum exploration, it is necessary to remove the background or the regional field, so that the anomalous field associated with the geological structures can be detailed. Thus, a mathematical representation of the Earth's field suitable for such applications is required. Such representations not only help extrapolate field values at locations where specific measurements have not been made, but they are also useful in assessing the possible mechanisms or sources of the main field.

Such a description and representation of the geomagnetic main field by a mathematical model is called a geomagnetic field model. Such field models are usually derived by fitting a spherical harmonic series to a set of global data, a method first put forth by Gauss in 1839. In such a spherical harmonic series representation of the field, usually the terms pertaining only to the sources interior to the Earth are considered, as the contributions from external sources are less than 1% and are lost in the overall accuracy of the fit of the model. Thus, these models are also referred to as internal field models.

With the advent of satellite measurements, quantitative modeling of external fields has also been attempted. However, such efforts have not been very successful primarily because of the difficulty in separating the internal and external fields from the measured data due to their relatively small amplitudes and extreme temporal and spatial variations. Nonetheless, external field models are becoming of increasing importance, as the accuracies of magnetic surveys have reached a point where a more accurate regional-residual separation in crustal anomalies studies is desired. Furthermore, since satellites are now being used to map the larger crustal anomalies, models of external fields are needed, as at satellite altitudes the external field effects are much more significant.

The use of such field models in many different areas of geomagnetic studies is manyfold and many such models, with different data bases, of different time validity and of different order and degree of spherical harmonic series expansion have been published in the literature since Gauss's first model in 1839. Many excellent discussions on field models themselves and comparative studies of various field models have also appeared in the literature in the recent years (Coles, 1979; Mead, 1979; Barraclough, 1978; Barraclough et al., 1978; Barraclough, 1976; Regan and Cain, 1975; Kane, 1973; and Cain, 1971). This report builds on such studies and presents a brief summary of the methods of determining field models and the current state-of-the-art in field model studies.

FIELD MODELS

Definition

A field model is basically a four dimensional function representing the geomagnetic field at any point in space and time. It is a function of colatitude ϕ, longitude Θ, geocentric distance r and time t. Since the geomagnetic field is derivable from a potential function, and because any potential could be expressed in terms of spherical harmonic series expansion, the field models themselves are usually expressed in terms of spherical harmonic series.

The scalar geomagnetic potential, V, at an external point (r, Θ, ϕ) arising from sources inside the earth could be expressed in terms of spherical harmonic series as (e.g., see Chapman and Bartels, 1940):

$$V = a \sum_{n=1}^{\infty} \left(\frac{a}{r}\right)^{n+1} \cdot \sum_{m=0}^{n} (g_n^m \cos m\phi + h_n^m \sin m\phi) P_n^m(\Theta), \qquad (1)$$

where

a	=	mean radius of the earth;
m,n	=	order, degree;
$P_n^m(\Theta)$	=	Schmidt's quasinormalized spherical functions; and
g_n^m, h_n^m	=	Gauss coefficients

The three orthogonal components of the field could be derived by taking the negative gradient of the potential. The northward, eastward, and downward components of the field are thus

$$\begin{aligned} X &= (1/r)\,(\partial V/\partial \Theta) \\ Y &= -(1/r \sin \Theta)\,(\partial V/\partial \phi) \qquad (2) \\ Z &= \partial V/\partial r \end{aligned}$$

Carrying out the differentiation of equation (1), the X, Y, Z components are given explicitly as

$$X = \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^{n+2} (g_n^m \cos m\phi + h_n^m \sin m\phi) \frac{\partial}{\partial\theta} P_n^m(\theta)$$

$$Y = \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^{n+2} (g_n^m \sin m\phi - h_n^m \cos m\phi) \quad P_n^m(\theta) \tag{3}$$

$$Z = \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^{n+2} (n+1) \; (g_n^m \cos m\phi + h_n^m \sin m\phi) \; P_n^m(\theta)$$

It is thus clear from the above expressions that if the set of Gauss coefficients are known in the series, the field components or any other element of the field may easily be computed at any point over the earth's surface. Such a set of Gauss coefficients are normally derived or adjusted by a least squares procedure, wherein some quantity such as the weighted sum of the squares of the difference between the value calculated from the model and the observed data is minimized; the sum being taken over a network of measurements covering the entire globe. It is to be noted here that if the global data set consists entirely of field components only, the set of Gauss coefficients can be derived by the least squares method in a straight-forward manner. If, however, the data set also consists of some other elements of the field, for example, scalar total field, F, then a non-linear least squares procedures must be used, as the resulting normal equations are non-linear in the Gauss coefficients. In such a case, the equations are quasi-normalized, and an assumed initial estimate of the set of coefficients are adjusted iteratively, until a fit to desired accuracy is achieved. Normally, this latter procedure is used in the field model calculations, since the global data set is invariably composed of other elements as well as the field components themselves.

In the spherical harmonic series representation of the geomagnetic potential (expression (1)), it is noted that the order, n of expansion should be carried out to infinity for a perfect theoretical representation. In practice, however, the expansion to such a high degree is neither possible nor warranted. Each value of n in the spherical harmonic expansion is a global wave number, i.e., each harmonic represents variations of the potential whose wavelengths are approximately 40,000/n km. Figure 6 shows this relationship graphically. It can be noted from this figure, that by carrying out the expansion to degrees higher than 8 to 10, the representation of smaller wavelength features in the model improves only very slowly. Also, as the degree n increases in the model, the number of coefficients increases rapidly as $[(n+1)^2 - 1]$. Hence, the number of calculations and the amount of associated computer time and storage used in the least-squares analysis increases greatly, approximately as the fourth power of n. Thus the series is truncated usually somewhere between the harmonic degree of 8 to 12, depending upon the accuracy of the data. Studies have indicated that field models of such degree and order are sufficient to model the field arising from the sources in the core of the earth. Some field models of even higher degree and order, however, have also been proposed.

A time parameter for modeling secular change can be readily introduced into the model by expanding the Gauss coefficients in a finite Taylor series about some mean time of the data set termed the epoch t_0:

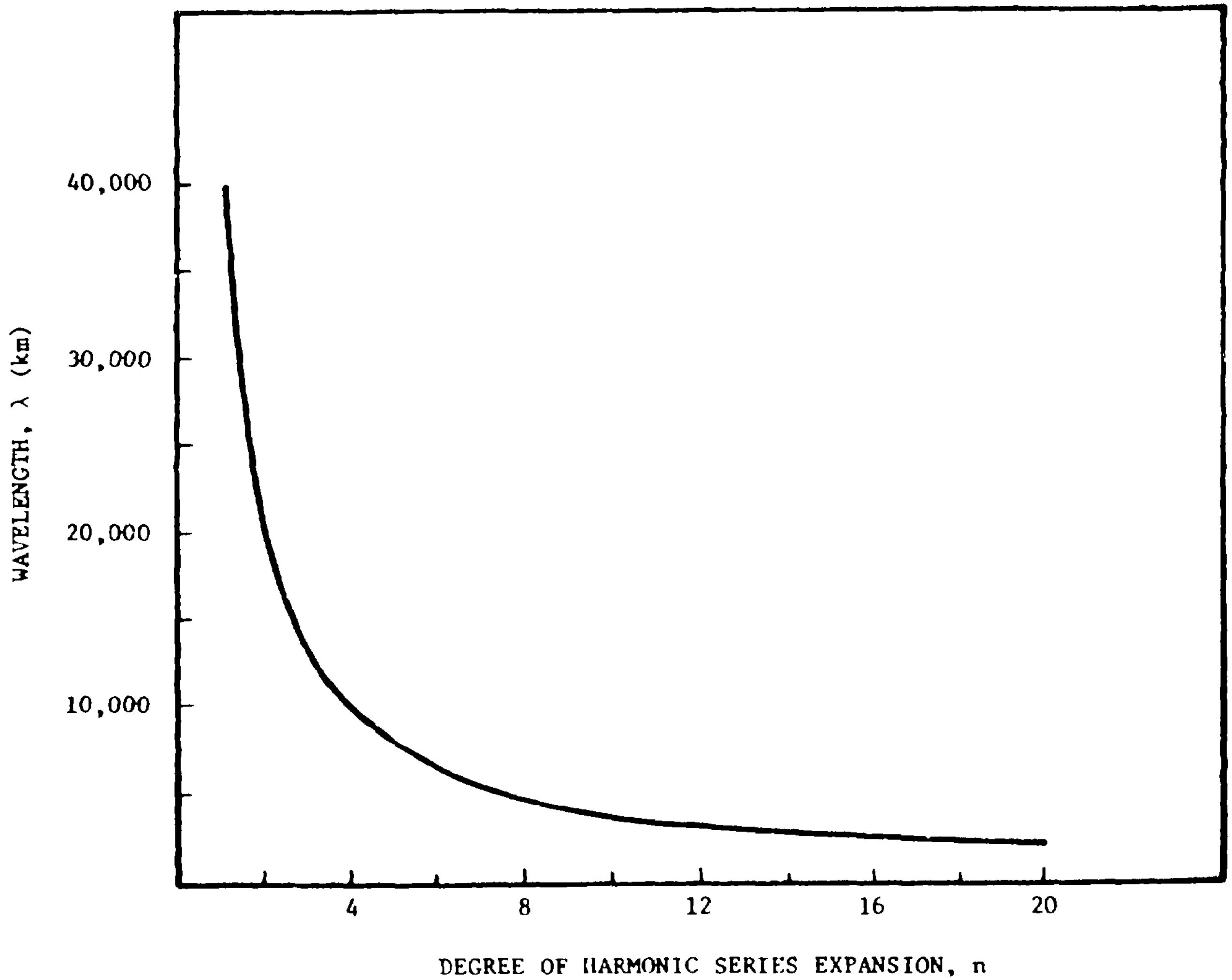

Figure 6. Appropriate relationship between the degree of spherical harmonic series expansion and the wavelength of the field represented in the model.

$$g = g_o + g'(t - t_o) + \frac{g''}{2}(t - t_o)^2 + \ldots$$

and (4)

$$h = h_o + h'(t - t_o) + \frac{h''}{2}(t - t_o)^2 + \ldots$$

Thus, the least squares fit is usually made in four dimensions (r, θ, ϕ, t) and the complete set of coefficients consists of the Gauss coefficients (g, h) and at least the first order secular change coefficients (g', h'). However, some models are derived by first independently estimating the secular change terms from the repeat stations or observatory data and then reducing the entire data set to a common epoch before least squares fit is made.

Practical Problems

It must be recognized that field models are essentially extrapolation functions, i.e., based on a set of observed data, they are used in predicting field values at locations over the global area where specific measurements have not been made. For such a prediction or extrapolation to be accurate the nature and limitations of the data base from which the model is derived must be fully examined and considered.

The first problem in accurate model determination comes from the fact that the available observed data are limited and irregularly distributed over the global area. Ideally, what would be desired is a uniform and dense coverage of data all over the globe. However, examination of charts of presently available land, sea and airborne magnetic survey coverage indicate that dense coverage is available in most parts of the globe, but there are still sizeable areas where the coverage is sparse or completely nil. Such irregular distribution of data prohibits the accurate determination of higher order harmonic coefficients in the model and may cause an aliasing effect in the lower order coefficients. It is one of the fundamental problems in spherical harmonic series field models that for calculation of each harmonic coefficient, knowledge of the field over the whole surface is required and little extra accuracy is gained locally by having a very dense local net of observations.

In relation to the effect of spatial distribution on the model calculation, the secular variation of the data or the temporal distribution of the data also must be considered. To obtain a sufficient amount of data distributed reasonably well over the global area, measurements obtained over a considerable period of time must be utilized. During such a period, the field values would have changed in a nonuniform and unpredictable manner owing to the secular variation of the main field. Thus, the model calculation would be in serious error. One way to compensate such error would be to reduce all the data to a common epoch by estimating the secular changes which occurred during the time interval of data. Such correct estimation, however, is made difficult as the magnetic observatories, where the secular changes are continuously recorded, are sparsely distributed over the globe and moreover, they are usually restricted to land masses leaving the vast ocean areas of the globe with no record of secular variations. The other way to compensate the effects of secular variation in the data is to incorporate the time term in the field model as previously mentioned, by expanding the Gauss coefficients in a finite Taylor series about some mean time in the data set and simultaneously solving for the time derivatives, called the secular change coefficients. In recent field models, this latter approach is preferred as the models then can not only be used for spatial predictions but for temporal predictions as well.

The problem of nonuniform global coverage was the driving force behind the initiation of Project MAGNET program by the U.S. Naval Oceanographic Office. This is an ongoing project to collect high altitude vector measurements over most of the free world. Although limited by inherent logistics of airborne operation this program has provided an invaluable data set for field model calculation. Two limitations of such a program, however, are the fact that it cannot obtain global coverage because of the political realities and that such coverage cannot be obtained in a short enough interval that the secular change remains constant. This problem has been somewhat minimized in the past decade with the aid of satellite measurements. Until recently, such data have only been of the scalar total field and as Stern and Bredekamp (1974) have noted, any model based on such data is very precise in representing the total field values at satellite altitudes but may be deficient in determining accurate vector values. The recent launch of MAGSAT satellite provides scalar as well as vector measurements over the globe at altitudes lower than earlier satellites and thus these data are expected to yield a better model for the current epoch. However, it must be noted that because of the lower resolution of satellite data, it does not replace the measurements from programs such as Project MAGNET. The two data sets are quite complementary.

Although the method of determining the coefficients on the field models is mathematically quite straightforward, the process of selecting, reducing and weighting of the data is not. Usually, the data base for the model calculation comprises diverse data sets, i.e., it may include data from airborne, land, marine, and satellite surveys as well as observatories. As each of the different data sets are subject to different kind of errors and have different resolution, their accuracies must be carefully estimated and the data carefully screened for spurious values. Usually, the data from observatories are given the highest weight because of their extreme accuracies. Such weighting and selection of the data ultimately affects the utility of a field model.

In addition to such practical concerns, the basic limitations of spherical harmonic series in representing the observed geomagnetic field accurately, must also be considered. As has been noted, the Earth's main field is composed of long wavelength anomalies arising from the core sources and small wavelength anomalies arising from sources in the crust. These long wavelength anomalies are modeled adequately by a spherical harmonic series expansion of degree and order ten, however, the small wavelength anomalies, which may be on the order of 10-100 km, are not. To model anomalies, whose wavelength are on the order of 10-100 km, would require expansions up to degree of 400 to 4000. The time and storage required for carrying out the spherical harmonic series to such a high degree would be prohibitively large, ruling out the use of spherical harmonic series to include crustal anomalies in its main field representation. Though this limitation of spherical harmonic series is not important in geophysical exploration applications, its impact for global studies and space science application is quite pronounced, since here a best representation of the total field is desired.

State-of-the-Art

The history of calculation and representation of Earth's main field by spherical harmonic series goes back more than a century since Guass first showed in 1839 that such a field model could conveniently be used to represent the field distribution over the Earth's surface. Since then many such field models have been proposed and published in the literature. Barraclough (1978) has recently published an exhaustive survey of over 264 field models that have appeared in the literature up to 1973 giving a detailed account of the methods and the data bases used in the different model derivations along with a complete list of the Gauss coefficients.

It must be recognized that the calculation of field models is an ongoing process, i.e., field models must be constantly updated, as more new data accumulates and older data become obsolete because of the unpredictive nature of the secular variation of the main field. Though the methodology of representing the main field by spherical harmonic series has remained the same over the years, two important advances in the method of calculation of field models should be noted. In the past, before the availability of modern digital computers, the coefficients of the harmonic series had to be computed by hand using numerical quadratures. This required availability of data distributed in a regular fashion in terms of latitude and longitude. Thus, the available survey and observatory data had to be plotted in a chart or map form and had to be interpolated to obtain a regular grid of data. Considerable smoothing and modification of original data was thus involved resulting in less accurate field models. Modern analysis, however, takes advantage of the power of digital computers and works directly on the set of irregularly distributed data, thus eliminating the intermediate stage of grid or map preparation. The other advancement in field model derivation, again with the help of digital computers, is the consideration of the oblateness of the earth which is important for the true representation of the field. Also, the computational capabilities of such modern computers facilitate the use of a larger data base and expansion of spherical harmonic series to a larger degree and order. Such improvements in field model definition have resulted in a better and more accurate representation of the earth's main field globally. For example, the best recent models are capable of predicting the average quiet-time field over most of the Earth's surface with an accuracy of about .2 degrees in direction and 200 gammas in magnitude whereas the older models could give at best one to two degrees in the directional accuracy. Another advancement in the method of representation of the main field by spherical harmonic series has been in the terms of analyzing the secular changes of the main field with secular acceleration terms now also being considered (Barraclough and Malin, 1979; Fougere, 1969; and Cain et al., 1967).

As previously mentioned, field models are usually computed to order and degree of about ten. This has been extended somewhat in recent years to twelve or thirteen as spectral studies (e.g., Cain, 1975) indicate that modeling of the core field requires such resolution. Even higher order models are desirable for some applications but the computer storage and time limitations associated with spherical harmonic analysis inhibit this. A solution to the excessive computer time and storage requirement of the conventional harmonic series analysis has recently been offered by The Analytical Sciences Corporation (TASC) group. Like the Fast Fourier Transform (FFT), they have proposed a fast computational scheme for the spherical harmonic expansion which seems to drastically reduce the time and storage requirements. The method, however, has a drawback in that a gridded set of data is required for the model construction, and only gridded data are computed from the model. In essence, the method involves expansion of the surface spherical harmonic in a Fourier series, which may then be evaluated using standard FFT technique.

Another approach to avoid the limited capability of present field models has been offered by Brown (1976). Unlike using the spherical harmonic representation as is conventionally done in field models, Brown has suggested use of Walsh functions to represent the data. These functions are binary and orthogonal, with a frequency characteristic similar to that of standard spherical harmonic functions. However, because of their binary nature, Walsh functions are more efficient and easy to compute on the present digital computers and thus, eliminate the practical limitations cited above for the spherical harmonic series. For magnetic field modeling, the Walsh functions would replace the Schmidt's quasinormalized spherical functions and the trigonometric functions in equation 1, retaining the harmonic term r^{-n-1} for analytic continuation of field values with altitude. Brown (1976) has estimated a computational time advantage of about 400,000 to 1 for Walsh functions over surface spherical harmonics of degree 125.

RECENT FIELD MODELS

As has been noted, geomagnetic field modeling is an ongoing and active area. Table 2 presents characteristics of some of the representative field models published in recent years and forms the basis for the following discussion. For details on earlier models, readers are referred to Barraclough's (1978) paper, where an exhaustive summary and listing of models published through 1973 are presented.

Of all the models listed in Table 2, the International Geomagnetic Reference Field (IGRF) model is probably the best known and most widely used model. It emerged primarily as a result of an international need to have a standard field model to which all the data should be referred or reduced to, thereby facilitating inter-comparison among different data sets. The first such model (IGRF65) was chosen at a symposium held by the International Association of Geomagnetism and Aeronomy (IAGA) in Washington, D.C. in 1968. The model was to represent the field at epoch 1965.0 with secular change coefficients that would extend its usefulness for at least a decade. Various groups of users were asked to present field models for consideration by the IAGA committee and the IGRF65 was developed as a compromise best model. Its main field coefficients are a weighted combination of four field models and the time terms are an average of five models. A detailed description of IGRF65 has been given by Cain and Cain (1971).

During the years of its use, however, the inherent weakness of the secular coefficient terms of the IGRF65 became rather evident, i.e., the predicted value from the model at a later time did not accurately represent the then measured field at many regions of the globe (e.g., Regan and Cain, 1975; Petkovic and Whitworth, 1975). Accordingly, resolutions passed at the IAGA General Assemblies

TABLE 2

MAIN CHARACTERISTICS OF SOME RECENT FIELD MODELS

	Epoch	Main Field n_{max}	Secular Terms n_{max}	Source Data	Data Interval
IGRF65	1965	8	8	Ground, airborne and satellite	
IGRF75	1975	8	8	Ground, airborne and satellite	
AWC70	1970	12	8	Ground, airborne and satellite (indirectly	1939-1970
AWC75	1975	12	8	Ground, airborne and satellite (indirectly)	1939-1974
POGO2/72	1968	13	13	Satellite	1965-1970
GSFC8/73	1970	14	14	Satellite, ground and airborne	1965-1971
IGS75	1975	12	8(SV) + 6(SA)	Ground, airborne and satellite (indirectly)	1955-1975
MGST3/80	1979-85	13		Satellite	1979
WC80	1980	12	8	Ground, airborne and satellite (indirectly)	1955-1980

in Kyoto 1973 and Grenoble 1975 resulted in the production of a revised model IGRF75, effective 1975.0. The IGRF75 retains the main field terms of IGRF65 updated to 1975.0 by means of the IGRF65 secular terms. Its new secular terms are the averages of the corresponding terms of the AWC75 and IGS75 models. Though a continuity in time was achieved by the use of secular change coefficients of IGRF65 in the production of IGRF75, the new model is not a good representative of the main field even at the epoch 1975.0 because of the inherent errors of the IGRF65 model. Accordingly a disclaimer was adopted at the IAGA meeting stating that the reference field is not intended as a source of compass information for nautical and aeronautical charts. Despite its poor representation of the main field, IGRF models are still very widely used in the reduction of geophysical survey data.

AWC70 (Hurwitz et al, 1974) and AWC75 (Peddie and Fabiano, 1976) models were derived primarily for the preparation of the American World Charts for the epoch 1970.0 and 1975.0 respectively. The main field terms in both are of maximum degree and order 12. The time terms were basically derived from observatory data, separately from the main field analyses, and have maximum degree 8.

The model IGS75 was developed for the preparation of British World Charts (Barraclough et al, 1975) for epoch 1975.0 and has main field coefficients of maximum degree 12. This model, in addition to secular variation terms also has secular acceleration terms.

The most recent model WC 80 (Barraclough and Barker, 1980) is the product of a joint effort by the United States and United Kingdon. It is to be used to construct the 1980 World Magnetic Delcination Charts published by the Defense Mapping Agency Hydrographic/Topographic Center in the U.S. and the British Hydrographic Department in the U.K. The main field coefficients are of degree and order 12, and the secular field coefficients are of degree and order 8.

Models POGO2/72 and GSFC8/73 have been derived primarily from the POGO series satellite data and have considerably higher main field and secular variation coefficient terms. The time terms in these models were determined simultaneously with the main field terms in the same least squares analysis.

Model MGST3/80 (Langel et al., 1980) is a preliminary model derived from just two days (November 5 and 6, 1979) of data from recently launched satellite, MAGSAT. The model is of particular interest, as it is the first time satellite-measured scalar and vector component data of the main field have been included in a model construction. A better field model from the MAGSAT satellite data, however, is expected to be derived shortly, as more data are processed and made available.

COMPARISON OF FIELD MODELS

The selection and use of any particular field model depends directly on the requirement of a user, for example, a user may select a model derived for the epoch 1965.0 if he has to reduce survey data collected during that time period, whereas in another case, a user may require a model derived for the latest epoch if he has to use it for the purpose of navigation. Difficulties arise, however, as there may be several models available for the same epoch and thus it becomes a tough choice to select a proper model.

Models are primarily judged on their prediction capabilities, both spatially and temporally. Many papers have appeared in the recent literature where the performance of a particular model and its comparison with other models have been evaluated in general terms. Such comparisons sometimes may be meaningless or misleading as the measured data against which the model are compared represent only a limited sample and may sometimes even be biased because of local long wavelength anomalies not represented in the field model. Moreover, a model judged superior to represent the field in a particular area may not be better or may even be worse in another region of the globe. Nonetheless results of such inter-comparisons provide valuable information about the general utility of a particular field model. Another criteria for comparison of spherical harmonic field models is their power spectra. By plotting the average degree variance of the models, defined as

$$P = \frac{1}{2n+1} \sum_{m=0}^{n} ({g_n^m}^2 + {h_n^m}^2) \qquad (5$$

one can infer the depth of the source mechanism for the various degree harmonics.

If these source depths do not change with time, then evaluation of the power of models at their various epochs should yield the same values. A model which does not agree with the concensus in source depth may be regarded with suspicion. Below, a brief summary of the results of intercomparison of some of the recent models and their power spectra are presented.

The deficiency and limitations of the International Geomagnetic Reference Field (IGRF), a most widely used model, are well reported in the literature (e.g., Regan and Cain, 1975). This model, as has been discussed earlier, is a composite of various models and was chosen initially for epoch 1965.0. Barraclough et al (1978) have recently published and presented another model for epoch 1965.0 which has been derived from a data set spanning the period of 1955 through 1975. As the mean of the data set correspond to the epoch 1965.0 of the new model, and more data could not be added to further refine it, they have referred to it as a definitive model for 1965.0. The authors have compared their definitive model against the IGRF65 model and the other candidate models from which IGRF65 was selected and averaged and have shown it to be considerably superior in representing the main field and its secular variations around the epoch of 1965.0. They noted that though main field coefficients of IGRF65 model represented the main field reasonably well at epoch of 1965.0, the poor secular term coefficients of the model severely limited its usefulness beyond 1965.0 They further showed that although IGRF65 model was considerably superior to other individual candidate models, its usefulness would have been considerably improved by a more judicious selection and proper weighting of different model coefficients than was actually done.

Mead (1979) has evaluated performance of four recent models namely AWC75 (Peddie and Fabiano, 1976), IGS75 (Barraclough et al., 1975), IGRF75

(IAGA, 1976) and POGO (8/71) (Langel, 1974), by comparing their predicted values against a set of observed annual mean data obtained from observatories around the globe. He calculated the residuals between the observatory annual means and the model predicted values for the three elements of the magnetic field namely B (total field), I (inclination) and D (declination) for the years 1973 through 1976. His results for the residuals ΔB, ΔI and $\Delta D \cos I$ in a histogram form are reproduced in Figures 7 through 9 respectively. As has long been discussed, the poorer quality of the IGRF model is clearly revealed here. The other three models can be judged as equally well in representing the main field with estimated accuracy on the order of 150-200 gammas and .2° in the amplitude and direction of the field respectively.

Coles (1969) has also made a similar comparative study of the prediction capabilities of several different recent models over regions of Canada. His study included seven different models (IGRF65, IGRF75, AWC70, AWC75, POGO2/72, GSFC8/73 and IGS75) and he considered data from observatories as well as from aeromagnetic surveys. His results comparing model predicted data with the observatory data indicate that in general such predictions are in better agreement with the portion of observatory data corresponding to the period of main data set used in the production of the field model. However, problems develop outside this time range mainly because of the non-linearity of the rate of secular change. In applying field models to the reduction of airborne survey data, his study clearly pointed out the advantage of models having higher order harmonic terms as they afford a much better spatial resolution needed in such regional-residual separation.

Langel et al. (1980) has recently proposed a new model (MGST 3/80) based on 2-days of MAGSAT satellite data. They have also used this set of data to

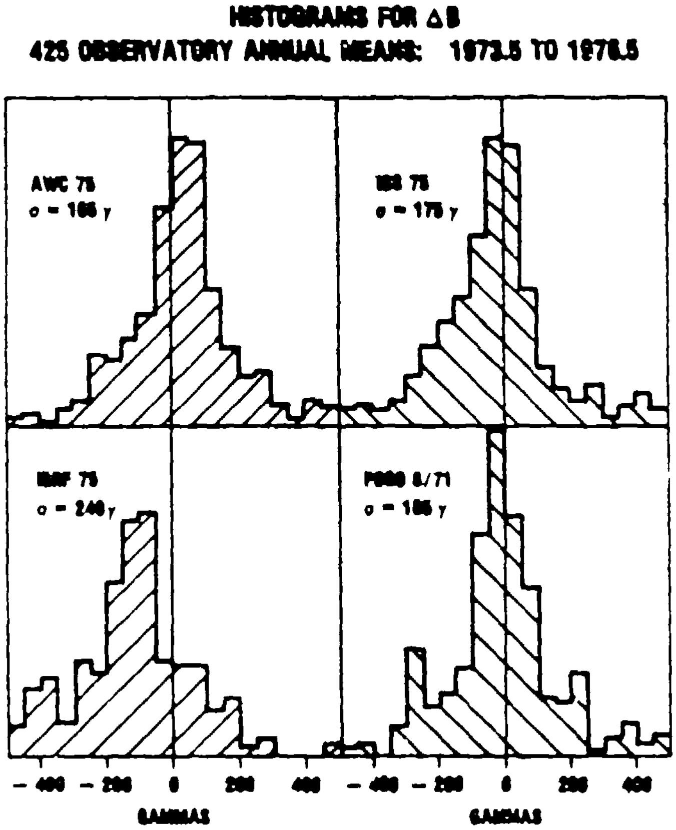

Figure 7. Histograms of the ΔB residuals for the four models. The error, σ, was taken to be half the width of the median 68% of the residual values (After Mead, 1979).

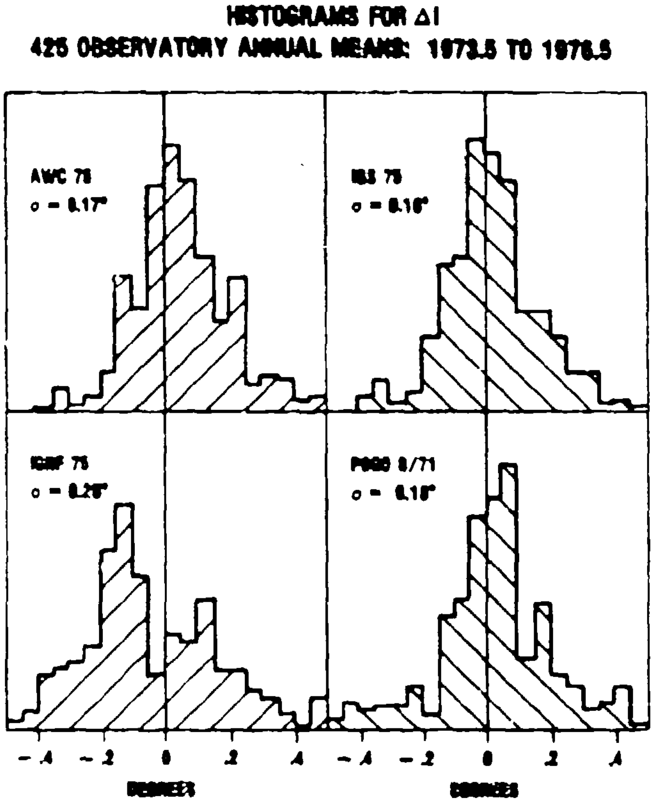

Figure 8. Histograms of ΔI residuals (After Mead, 1979).

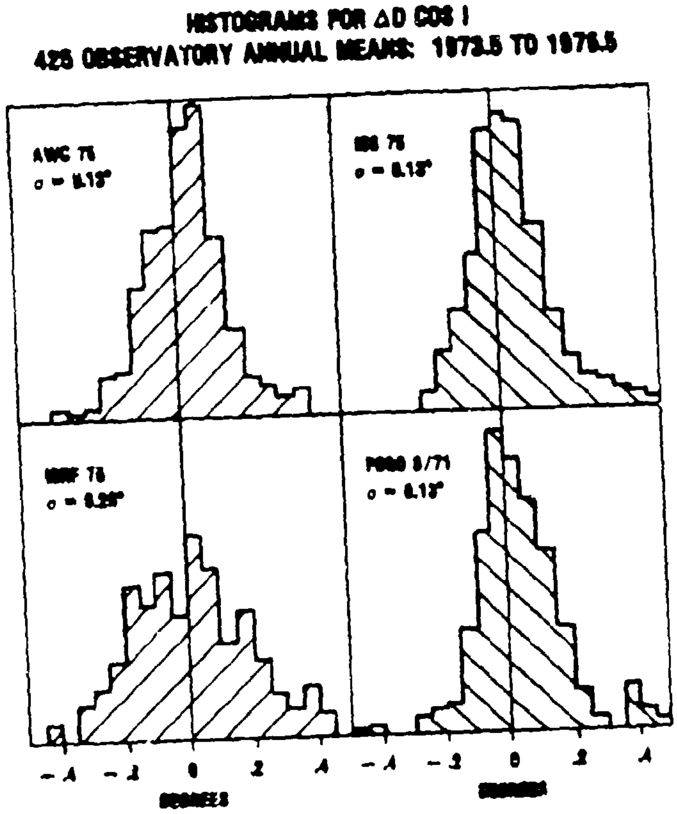

Figure 9. Histograms of ΔD cosI (After Mead, 1979).

compare the prediction capabilities of other recent models at the MAGSAT altitude and epoch. Table 3 summarily presents the results of their comparative study. The results clearly point out that POGO2/72 model which was based only on scalar measurements made by earlier POGO satellites, is the best model in correctly predicting the scalar field measured by MAGSAT. However, AWC75 and IGS75 appear to better describe the current vector fields.

Comparison of the power spectra of seven spherical harmonic models was carried out following the procedure of Cain (1975). The power of a model was evaluated using equation (5) and the epoch value of the model coefficients. The power spectra values for the POGO6/74, POGO2/72, AWC75, IGRF75, GSFC12/66, IGRF10/68, and the WC80 models are displayed in Figure 10, along with an approximate power law curve fit adapted from Cain (1975). The different slopes of the straight-line segments of this curve are interpreted by Cain as representative of the core ($n<8$), crust ($n>13$) and mantle transition ($8<n<13$) source region.

All of the models yield virtually the same powers for $n<8$, and even for $n=10$, no significant disagreements exist. However, for $n=11$ and 12, two models WC80 and POGO2/72 yield significantly higher power than POGO6/74 and AWC75. This may be due to the effects of aliasing, or truncation error, since the higher degree POGO6/74 model has much lower power at $n=12$. The sudden increase in power at $n=13$ for POGO2/72 is especially suspicious. Since the WC80 and POGO6/71 models both have epochs which are many years away from those of the POGO6/74 or AWC75 models, it is possible that the observed differences in power are somehow due to secular changes in the depth of the source mechanisms. This question may be resolved by study of additional models having common epochs.

TABLE 3

Residuals (in gammas) of selected MAGSAT data to some published field models. (After Langel et al., 1980)

	IGRF 1975	AWC/75	IGS/75	POGO(2/72)	MGST(3/80)
Scalar:mean deviation	-90	61	23	9	0
:standard deviation	125	127	120	107	8
B_r:mean deviation	29	46	40	25	21
:standard deviation	204	153	137	211	44
B_θ:mean deviation	44	-10	8	12	12
:standard deviation	146	115	114	145	107
B_ϕ:mean deviation	62	62	61	61	62
:standard deviation	181	157	155	208	129

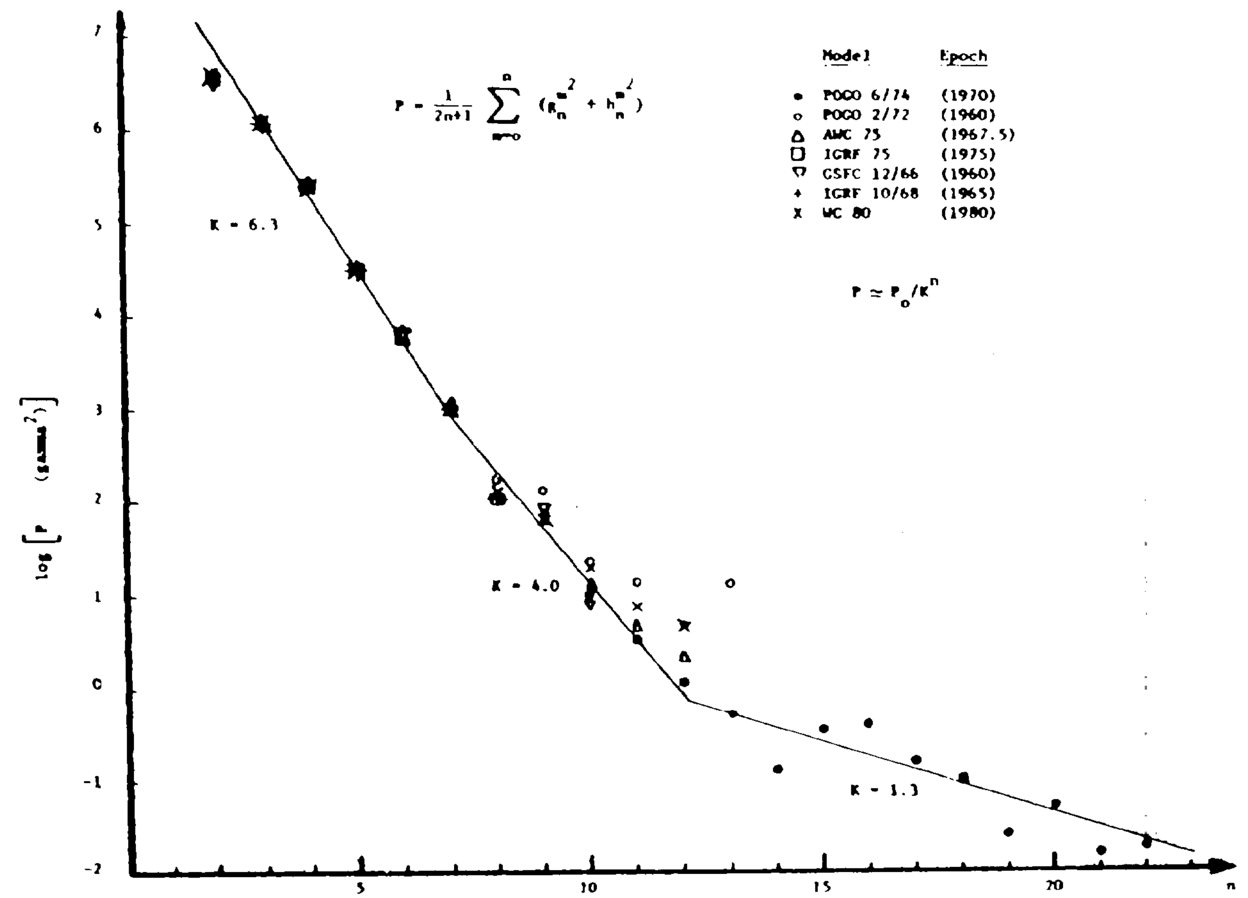

Figure 10. Normalized power in spherical harmonics of internal field models evaluated at the appropriate epoch. The power law curve fit is taken from Cain (1975).

SUMMARY AND CONCLUSIONS

The Earth's magnetic field is complex and dynamically varying with approximately 99% of the field at the Earth's surface originating from internal sources and the remainder from sources outside the Earth, mainly in the ionosphere and magnetosphere. That part originating from internal sources is called the internal field or the main field and that originating from external sources referred to as the external field. The main field varies relatively smoothly over the Earth's surface and is approximately dipolar in nature. The main field, however, is not permanent in time but rather varies relatively slowly over period of years and such variation in the main field is termed secular variation.

From measurements of the field over the Earth's surface, mathematical models of the main field and its secular variation can be derived which describe the measured field. Such a mathematical representation of the global main field is usually done in terms of a truncated spherical harmonic series and is called a geomagnetic field model or simply field model. Such models are essentially data extrapolation functions, i.e., from a set of measured global data they are used in to extrapolate field values both spatially and temporally. This analytical representation of the global main field makes field models ideally suited for data processing and are used in a variety of applications including preparation of world magnetic charts for navigational purposes, removal of regional trends from magnetic surveys to isolate local crustal anomalies, and in numerous space science studies.

Comparison and evaluation of recent field models indicate that the best field models could describe the field with overall accuracies better than approximately 150-200 gammas in magnitude and .2° in direction of the field. Effects of crustal sources as well as the external sources are not included in the field models due to their limited spatial and temporal frequencies. This may be a limitation of current field models and continued improvement in accurate representation of the Earth's field would require inclusion of these two sources, namely, the crustal sources and the external sources in the field models. Inclusion of crustal anomalies in the model, however, is neither warranted nor desired for geophysical applications but may be highly desirable for other applications such as navigation and space science studies. Inclusion of such a signal would require carrying out the spherical harmonic series analysis to a much higher degree and order than done at present. This is a formidable task, not only because of the immensely increased computer time and storage requirement but also because the necessary detailed global spatial data are not available. A better solution might be to add local function representations of the crustal anomalies of a particular region to the global field model, which then would allow a much better detailed definition of the field over that particular region.

Another problem in accurate representation of the geomagnetic field by field models comes from the fact that the Earth's field is in a continual state of change. Such secular change must be monitored and retained in the model, if model prediction is to be valid over future years. Lack of enough data has generally prevented accurate global modeling of secular changes and only first order linear time variation terms over the period of the data base are calculated and retained in current models. Few models containing second order time terms (secular acceleration) have been calculated. Such second degree time terms may in certain instances improve the description, but can also worsen the predictive capability of a model if changes in the observed acceleration occur. Thus frequent updating of field models is necessary to account for the secular changes in the Earth's main field.

Mapping of Earth's magnetic field by satellites in the past decade have considerably aided in better representation of the field as they provided essentially global coverage over a very short period of time. In the past, such data were limited to the total field measurements only and as Stern and Bredekamp (1974) have noted the models based on these measurements may be very precise in representing total field values at satellite altitudes, but they may be deficient in determining accurate vector values. Such deficiencies, however, are expected to be removed from the analysis of MAGSAT satellite data, which is to provide vector field measurements as well as the total field values. Despite such satellite data, the role of airborne survey data like Project MAGNET data in defining the Earth's field cannot be denied as the two types of data are essentially complimentary to each other and analysis of combined data sets result in a much better model of the Earth's field.

REFERENCES

Barker, F.S. and Barraclough, D.R., 1980, "World Magnetic Chart Model for 1980," EOS, Trans. Am. Geophys. Union, 61, 453.

Barraclough, D.R., 1978, "Spherical harmonic models of the geomagnetic field," Institute of Geological Sciences, Geomagnetic Bull, No. 8.

Barraclough, D.R., 1976, "Spherical harmonic analysis of the geomagnetic secular variaion--a review of methods," Phys. Earth Planet. Inst., 12, 365-382.

Barraclough, D.R. and Barker, F.S., 1980, "World Magnetic Chart Model for 1980," EOS, Trans. Am. Geophys. Union, 61, 453.

Barraclough, D.R., Harwood, J.M., Leaton, B.R., and Malin, S.R.C., 1978, "A definitive model of the geomagnetic field and its secular variation for 1965--I. Derivation of model and comparison with the IGRF," Geophys. J.R. Astr. Soc., 55, 111-121.

Barraclough, D.R., Harwood, J.M., Leaton, B.R., and Malin, S.R.C., 1975, "A model of the geomagnetic field at epoch 1975 [IGS/75]," Geophys. J.R. Astr. Soc., 43, 645-659.

Barraclough, D.R. and Malin, S.R.C., 1978, "Geomagnetic secular acceleration," Geophys. J.R. Astr. Soc., 58, 785-793.

Brown, R.D., 1976, "Geophysical data analysis using binary sample functions," Physics of the Earth and Planetary Interiors, 12, 253-267.

Cain, J.C., 1975, "Structure and secular change of the geomagnetic field," Rev. Geophys. Space Phys., 13, 203-206, U.S. Nat. Rep. to IUGG.

Cain, J.C., 1971, "Geomagnetic models from satellite surveys," Rev. Geophys. Space Phys., 9, 259-273.

Cain, J.C. and Cain, S.J., 1971, "Derivation of the International Geomagnetic Reference Field," NASA Technical Note D-6237, 1-35.

Cain, J.C., Hendricks, S.J., Langel, R.A., and Hudson, W.V., 1967, "A proposed model for the International Geomagnetic Reference Field, 1965," J. Geomag. Geoelectr., 19, 335-355.

Chapman, S. and Bartels, J., 1940, "Geomagnetism," Vols. I and II, 1049 pp., Clarendon Press, London.

Coles, R.L., 1979, "Some comparisons among geomagnetic field models, observatory data and airborne magnetometer data: Implications for broad scale anomaly studies over Canada," J. Geomag. Geoelectr., 31, 459-478.

REFERENCES (Cont'd)

Fougere, P.F., 1969, "Spherical harmonic analysis, 3. The Earth's magnetic field 1900-1965," J. Geomag. Geoelectr., Kyoto, 21, 685-695.

Gauss, C.F., 1839, "Allgemeine Theorie des Erdmagnetismus," [General theory of terrestrial magnetism]. Resulate aus den Beobachtungen des magnetischen Verein im Jahre 1838. (Leipzi.) [In German.]; and 1841. Pp. 184-251 in Scientific memoirs selected from the transactions of foreign academies and learned societies and from foreign journals, Vol. 2. Taylor, R. (Editor). [Translation into English by Mrs. Sabine, revised by Sir John Herschel.]; see also pp. 119-193 in Gauss, C.F. 1877. Werke, Vol. 5. (Göttingen; Königlichen Geselschaft der Wissenschaften.).

Hurwitz, L., Fabiano, E.B., and Peddie, N.W., 1974, "A model of the geomagnetic field for 1970," J. Geophys. Res., 79, 1716-1717.

IAGA Division I Study Group, 1976, "International Geomagnetic Reference Field 1975 [IGRF 1975]," EOS, Trans. Am. Geophys. Union, 57, 120-121.

Kane, M.F., 1973, "The reference field in regional and local magnetic investigations," Intl. Assoc. Geomag. and Aeron., Bull. No. 34, p. 274.

Langel, R.A., Estes, R.H., Mead, G.D., Fabiano, E.B., and Lancester, E.R., 1980, "Initial Geomagnetic Field Model from MAGSAT," Submitted to Geophysical Research Letters.

Malin, S.R.C., 1969, "Geomagnetic secular variation and its changes, 1942·5 to 1962·5," Geophys. J.R. Astro. Soc., 17, 415-441.

Mead, G.D., 1979, "An Evaluation of Recent Field Models, in Quantitative Modelling of Magnetospheric Processes," W.P. Olson, Editor, Geophysical Monograph 21, 110-117, American Geophysical Union.

Patkovic, J.J. and Whitworth, R., 1975, "Problems in secular variation in the Australian region," EOS, Trans. Am. Geophys. Union, 56, 547 (abstract only).

Peddie, N.W. and Fabiano, E.B., 1976, "A model of the geomagnetic field for 1975 [AWC/75]," J. Geophys. Res., 81, 2539-2542.

Regan, R.D. and Cain, J.C., 1975, "The use of geomagnetic field models in magnetic surveys," Geophys., 40, 621-629.

Stern, D.P. and Bredekamp, J., 1974, "Error enhancement in geomagnetic model derived from scalar data," Goddard Space Flight Center Tech. Rep., X-602-74-45.